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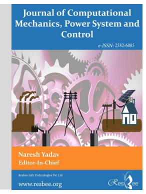

Design Optimization of a Hybrid Solar PV Panel and Pumped Hydro Energy Supply System

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Abstract: Advancements in technology unveil a myriad of electrical and electronic breakthroughs geared towards efficiently harnessing limited resources to meet human energy demands. The optimization of hybrid solar PV panels and pumped hydro energy supply systems plays a pivotal role in utilizing natural resources effectively. This initiative not only benefits humanity but also fosters environmental sustainability. The study investigated the design optimization of these hybrid systems, focusing on understanding solar radiation patterns, identifying geographical influences on solar radiation, formulating a mathematical model for system optimization, and determining the optimal configuration of PV panels and pumped hydro storage. Through a comparative analysis approach and eight weeks of data collection, the study addressed key research questions related to solar radiation patterns and optimal system design. The findings highlighted regions with heightened solar radiation levels, showcasing substantial potential for power generation and emphasizing the system's efficiency. Optimizing system design significantly boosted power generation, promoted renewable energy utilization, and enhanced energy storage capacity. The study underscored the benefits of optimizing hybrid solar PV panels and pumped hydro energy supply systems for sustainable energy usage. Optimizing the design of solar PV panels and pumped hydro energy supply systems as examined across diverse climatic conditions in a developing country, not only enhances power generation but also improves the integration of renewable energy sources and boosts energy storage capacities, particularly beneficial for less economically prosperous regions. Additionally, the study provides valuable insights for advancing energy research in economically viable areas. Recommendations included conducting site-specific assessments, utilizing advanced modeling tools, implementing regular maintenance protocols, and enhancing communication among system components.

Keywords: Hybrid Solar Pv Panel, Pumped Hydro, Energy Optimization, Renewable Energy System Design.

Nomenclature

Abbreviations	Descriptions
PV	Photovoltaic
PHES	Pumped Hydro Energy Storage
Gsc	Solar Constant
cos(θ)	Cosine of the Solar Zenith Angle
cos(β)	Cosine of the Tilt Angle of the Solar Panel Or the Angle of Inclination from the Horizontal Plane.
AM	Air Mass Coefficient
CMF	Cloud Modification Factor
R_S	Series Resistance
R_{sh}	Shunt/Parallel Resistance
Voc	Voltage Source
H_{out}	Heights of Water in the Outlet Tanks
H_{in}	Heights of Water in the Inlet Tanks
W/m²	Watts Per Square Meter
Z_{phs}	Impedance for the Phase.
R_g	Resistance of the Transformer
X_m	Magnetizing Reactance
Ohms	Unit of Electrical Resistance
MPPT	Maximum Power Point
kWh	Kilowatt-Hour
Vmp	Maximum Power Point Voltage

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1. Background to the Study

In contemporary times, there has been a discernible surge in the fervor surrounding solar-driven water pumping systems, particularly in the context of utilizing renewable energy sources for power generation [8]. Additionally, the incorporation of these systems offers substantial advantages to countries strategically positioned close to the equator such as Nigeria [25], thereby fostering a sustainable and environmentally conscious energy supply. These systems consist of a pump propelled by a PV system and a hydroelectric turbine that transforms stored water into electricity [24]. The amalgamation of these technologies holds the potential to facilitate the effective utilization of renewable energy sources. Nevertheless, there exists a pressing necessity for optimization. Beyond the renewable facet, the pivotal issue of adaptation arises. The adaptability component underscores the imperative to scrutinize and enhance this innovation through the prism of optimization to avert it from becoming yet another defunct electrical engineering concept relegated to the annals of history. When delving into the realm of designing a hybrid solar PV and pumped hydro system, optimization entails the maximization of the system's efficiency and performance by meticulously considering a myriad of electrical engineering factors [6] and juxtaposing them against suboptimal designs.

Efficiency in this context refers to the system's ability to convert available energy into usable electrical power effectively [1]. For example, optimizing the design of solar PV panels entails selecting high-efficiency panels that can convert a greater percentage of solar energy into electrical energy. In their study on solar-powered water pumping systems, [22] emphasize the importance of accurately estimating the tank volume to ensure an adequate water supply. They stress that an optimized system should take into account factors such as water demand, solar availability, and seasonal variations in water availability. By optimizing the tank volume, surplus water can be stored during periods of high solar availability and meet water demand during low solar availability. Similarly, optimizing the design of the hydroelectric turbine involves selecting a turbine with high efficiency to maximize the conversion of water flow into electrical power [13].

Performance, on the other hand, pertains to the overall effectiveness and reliability of the system in meeting the desired energy demands [3]. Therefore, the pursuit of optimization aims to enhance the system's performance by considering factors such as solar radiation patterns, energy demand, geographical features, energy losses, system stability, power output, and impedance matching. For instance, optimizing the energy requirements of the pump involves selecting a pump with high performance and efficiency to minimize energy wastage and ensure a stable water supply for continuous and reliable power provision. Through this study, it is anticipated that the findings will contribute to the sustainable development of renewable energy systems in Nigeria and beyond.

1.1 Statement of the Problem

The enduring energy crisis in Nigeria poses a formidable challenge impacting various sectors of the economy due to antiquated technologies, deficient maintenance, and constrained capacity within the current power generation infrastructure. To address this issue effectively, the exploration of sustainable energy alternatives like hybrid solar photovoltaic (PV) panels and pumped hydro energy supply systems is imperative to mitigate the energy shortfall. While past inquiries have scrutinized solar PV systems and pumped hydro energy storage separately, limited attention has been paid to optimizing the design of a hybrid system amalgamating both technologies within the Nigerian context. Existing studies have predominantly focused on theoretical frameworks or case analyses from foreign nations, potentially inadequately reflecting the Nigerian energy scenario. Thus, further research is indispensable to refine the design, specifically tailored to align with the distinctive solar radiation patterns, geographical features, and energy consumption patterns in Nigeria. This aspect constitutes the focal point of the investigation.

1.2 Aim and Objectives of the Study

This study was aimed at the design optimization of a hybrid solar PV panel and pumped hydro energy supply system in Nigeria. Specifically, the objectives were to:

1. Ascertain the solar radiation patterns in selected northern and southern states of Nigeria.
2. Highlight the geographical features that impact solar radiation in these states.
3. Develop a mathematical model for optimizing the design parameters of the hybrid system, considering factors such as panel tilt angle, tank volume, inlet and outlet flow rates, total dynamic head, pump and motor size, storage capacity, power generation calculations, storage capacity, and system sizing.

4. Determine the configuration of PV panels and pumped hydro storage that provides the best performance in terms of energy generation, storage capacity, and impedance matching.

1.3 Research Questions

1. How does solar radiation vary across the selected northern and southern states of Nigeria?
2. What are the geographical features that impact solar radiation in these states?
3. How can the design parameters of this hybrid system, including panel tilt angle, tank volume, inlet, and outlet flow rates, total dynamic head, pump and motor size, storage capacity, power generation calculations, storage capacity, and system sizing, be optimized through the development of a mathematical model?
4. What configuration of PV panels and pumped hydro storage would optimize energy generation, storage capacity, and impedance matching for improved performance?

The rest of the paper is explained as follows: Section 2 explains the Literature review, Section 3 elaborates on the methods and methodology, Section 4 mentions the result and discussion, Section 5 covers the advantages and disadvantages, Section 6 concludes the paper, and Section 7 mentions the recommendation.

2. Literature Review

Design optimization is the art of fine-tuning a system to achieve superior performance within specified limits, as [18] rightly elucidates, by striking a balance between the highest efficiency and the lowest expenditure. In this vein, the harmonization of renewable energy systems, particularly the amalgamation of PV and pumped hydro storage, has become imperative, as [16] acknowledges in their discussion of sustainable development. Solar PV panels offer a direct method of converting sunlight to electricity through the photovoltaic effect, posited by [24] to hold substantial promise for global energy needs. Despite their potential, [30] admits that the fluctuating nature of solar output - reliant on weather and daylight - poses a significant dilemma for achieving a consistent energy supply. In contrast, the integration of Solar PV with pumped hydro storage systems counteracts such variability [24] [31] and corroborates that PHES is a robust strategy to address the intermittency challenge of solar energy.

During excess generation, solar power is used to pump water to higher elevation storage, and when the demand surges or sunlight is scarce, the water is released to generate hydroelectricity, as [26] describes. Design optimization of this hybrid system thus demands an intricate approach of impedance matching [32] [33] with computational models [23] [5] assessing the performance across diverse scenarios [21][7] to determine an optimal set-up for harmonizing energy generation and consumption [12] [27]. It goes in line with the research by [28], emphasizing the need to methodically analyze solar array sizing, pump and turbine capacities, and reservoir volume. The locational analysis, as [14] concurs with [20] as well as [19], is another critical facet of optimization, where the placement of solar arrays will not only seek maximal solar irradiance but also align with the geographical prerequisites for effective PHES deployment.

In the same vein, [9] note that the hybrid system's true strength lies in the PV panels' capacity to yield a predictable power supply, which is advantageously complemented by the pumped hydro system's ability to rapidly respond to any discrepancies. With optimized design, operational strategies can be crafted, as [17] in agreement with [15], to intelligently manage energy according to varying solar outputs, grid demands, and market conditions. In line with the insights offered by [34] and corroborated by [4], optimizing the design of a hybrid Solar PV and pumped hydro energy supply system is a forward-looking venture in renewable energy technology. By acknowledging the collective advantages and limitations of solar and hydro technologies, designers can craft systems that not only meet energy requirements but are economically and environmentally sustainable, projecting society toward a greener and more resilient energy future.

2.1Theoretical Framework

The buck-boost converter is one theory that is very suitable in guiding this study. Tested by Engineer Slobodan Cuk [10] under the supervision of Professor R. D. Middle brook in 1977 emphasizes the necessity of an adept electrical converter design capable of delivering regulated power to a load by efficiently inverting the polarity of the output voltage relative to the input [10]. This feature makes the buck-boost converter very versatile, as it can handle inputs that are either higher or lower than the desired output voltage. Stepping down a higher input voltage to a lower output voltage. A buck-boost converter design reduces the output voltage while amplifying the output current [29], proving indispensable in the quest to optimize power transfer through impedance matching. As observed in the works of [2], the functionality of the buck-boost converter design hinges on the principles of energy transfer in inductors and capacitors, switch-duty cycles, ripple characteristics, and efficiency optimizations. The hybrid system's efficiency largely depends on how effectively power is transferred from the solar panels to the energy storage system and then to the load [24]. The buck-boost converter helps maintain the desired voltage level irrespective of the variations in solar power output, thereby ensuring efficient power transfer [10]. In a pumped hydro storage system, there may be times when the energy needs to be both stored (pumping water uphill) and retrieved (flowing water downhill) depending on the grid demand. A buck-boost converter allows for flexibility in such a control strategy by providing the ability to manipulate the voltage in either direction while also maintaining an inverted output [11], when necessary. Additionally, impedance matching is essential to make this transfer with minimal losses; the converter aids in matching the impedance of the source (solar PV or hydro generator) with the load.

2.2 Review Table

Table 1. *Related Studies on Design Optimization of Hybridized Power Generators*

Authors	Main Objective	Methodology	Key Findings	Contributions to Knowledge
Stieglitz and Platzer [30]	Analyze solar radiation	Literature review	Understanding solar radiation patterns	Insight into solar radiation for system design
Pali and Vadhera [26]	Design a solar PV system with pumped-water storage	Simulation and analysis	Novel solar PV system with storage	Improved energy generation and reliability
Tan *et al.* [31]	Review energy storage for smart grid	Review and analysis	Comprehensive overview of energy storage	Advancement in smart grid energy systems
Elkadeem *et al.* [14]	Optimize hybrid renewable energy system	Geospatial multi-criteria analysis	Sustainable design and optimal siting	Improved decision-making in renewable systems
Ruhnau and Qvist [28]	Study storage requirements in renewable electricity systems	Modeling and analysis	Understanding storage needs in renewable systems	Contributing to 100% renewable systems
Zisos *et al.*[33]	Design optimization of hybrid renewable and pumped hydropower systems	Optimization and uncertainty analysis	Efficient design under uncertainty	Insight into hybrid renewable and pumped hydropower systems
Wang *et al.* [32]	Implement maximum power point tracking for a micro hydropower generator	Experimental and analysis	Improved power generation efficiency	Enhanced performance of micro hydropower systems

2.3 Identified Research Gap

The literature review revealed that while several studies have explored solar and hydropower systems individually, limited research exists on hybridizing solar PV panels with pumped hydro energy storage specifically for the Nigerian context. Most optimization studies of such hybrid systems focused on temperate regions with limited consideration for the unique climatic conditions and varying solar irradiance levels across Nigeria. Therefore, a research gap exists to conduct a comprehensive optimization of hybrid solar-pumped hydro system designs tailored for the climatic diversity within Nigeria. Addressing this gap could help advance the deployment of renewable hybrid solutions to expand energy access in the country.

3. Materials and Methodology

The successful evaluation of any renewable energy system requires that appropriate criteria of the chosen site be considered. Details are focused on the calculations of solar radiation in selected locations, geographical features, panel tilt angle, tank volume, inlet and outlet flow rates, total dynamic head, pump, and motorsize as well as storage capacity calculations were estimated in 8 weeks between December and January.

3.1 Mathematical Equations for Solar Radiation Pattern:

The solar radiation incident on a surface (G) was calculated using the following equation:

$$G = Gsc \ * \ cos(\theta) \ * \ cos(\beta) \ * \ \tau \tag{1}$$

Where

Gsc = Solar constant (approximately 1361 W/m²),

θ=Solar zenith angle,

β =Tilt angle of the solar panel,

τ =Transmittance factor.

3.2 Mathematical Equations for the Geographical Features

Impact of latitude: The angle of incidence of solar radiation is influenced by the latitude of a location. The solar zenith angle (θ) was calculated using the latitude (φ) and the declination angle (δ) of the sun between December and January:

$$\theta = sin^{\wedge}(-1)(sin(\varphi) * sin(\delta) + cos(\varphi) * cos(\delta) * cos(h)) \tag{2}$$

Where

θ = Solar zenith angle

φ = Latitude of the location

δ = Declination angle of the sun

Impact of altitude: Altitude affects solar radiation by influencing air mass and atmospheric absorption. The air mass coefficient (AM) was calculated using the altitude (h) above sea level:

$$AM = \frac{1}{\mathrm{Sin}(\theta_{Zenith})} = \frac{1}{\mathrm{Cos}\left(90^{\circ} - (\theta_{Zenith} - h)\right)} \tag{3}$$

Where

AM = Air mass coefficient

θ_{Zenith} = Solar zenith angle

h = Altitude above sea level

Impact of cloud cover: Cloud cover significantly affects the amount of solar radiation reaching the Earth's surface. The CMF can be used to adjust for the reduction in solar radiation due to clouds:

$$I_{actual} = I_{clear} * CMF \tag{4}$$

Where

I_{actual} = Actual solar radiation reaching the surface

I_{clear} = Clear-sky solar radiation

CMF = Cloud modification factor

Impact of atmospheric conditions: Atmospheric conditions like aerosols and water vapor content can scatter and absorb incoming solar radiation. The atmospheric transmittance factor (τ_{α}) accounts for these effects:

$$I_{actual} = I_{extraterrestrial} * \tau_{\alpha} \tag{5}$$

Where

I_{actual} = Actual solar radiation reaching the surface

$I_{extraterrestrial}$ = Extraterrestrial solar radiation

τ_{α} = Atmospheric transmittance factor

The τ_{α} value considers factors like Rayleigh scattering, Mie scattering from aerosols, and absorption by water vapor.

3.3 Mathematical Model for Optimizing the Hybrid System Design Parameters

Solar declination angle (δ): This angle represents the position of the sun relative to the Earth's equator and varies throughout the year. It is calculated based on the day of the year using the formula:

$$\delta = 23.45\,\mathrm{Sin}\left[\frac{360}{365}(d-81)\right] \quad (6)$$

Solar elevation angle (α): The solar elevation angle is the angle between the horizontal plane and the line of sight to the sun. It is calculated based on the latitude (φ), solar declination angle (δ), and the hour angle (h) using the formula:

$$\alpha = \arcsin\left(\sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\cos(h)\right) \quad (7)$$

Incident solar radiation on the tilted surface (Gt): This equation calculates the amount of solar radiation that falls on a tilted surface, taking into account the beam radiation (Gb) and the tilt angle (θ). It is given by:

$$Gt = Gb\cos(\theta) \quad (8)$$

The power generated by the solar PV panels can be calculated as:

$$P = A * \eta * G \quad (9)$$

Where
A = Surface area of the solar panels,
η = Efficiency of the solar panels,
G = Solar radiation incident on the panels.
Generally, the formula for calculating optimal tilt angle is given by:

$$Tilt\ angle = Latitude + seasonal\ offset \quad (10)$$

Where
Latitude= The angle of the location from the equator.
Seasonal offset= An additional angle to adjust for the season.
Tank volume calculation:

$$V_{\tan k} = \frac{\pi d^2}{4} * h_s \quad (11)$$

Where:
d = Diameter of tank
h_s = Vertical rise
Inlet flow rate of water calculation:

$$Q_{inlet} = \frac{V_{\tan k}}{t} \quad (12)$$

Where
$V_{\tan k}$ =Volume of tank
t = Time taken for pumping water
Outlet flow rate of water calculation:

$$Q_{outlet} = \frac{V_{\tan k}}{t} \quad (13)$$

Where:
t = Time taken for discharging water
Total dynamic head (TDH)calculation:

$$THD = h_p + h_s + F_l \quad (14)$$

$$F_l = \left[L_t + \Sigma\left(n_f * f_e\right)\right] * \frac{f_h}{100} \quad (15)$$

Where
h_p =Pumping level

h_s =Vertical rise

F_l = Frictional loss in pipe

L_t = Total length of pipe

f_e = Fittings of the pipe, in feet (these come in standard values, and it depends on pipe diameter on pipe's diameter)

n_f = Number of similar fittings in the system

f_h =Friction loss of head per 100 feet of pipe, depending on the pipe's diameter and flow rate

Pump And Motor Size Calculation:

Pump hydraulic power

$$P_h = \frac{Q * \rho g * THD}{1000} \tag{16}$$

Where

Q =Flow rate m^3/s

ρ =Density of water kg/m^3

g = Acceleration due to gravity in m/s

THD =Total dynamic head in m

Induction motor size

$$P_m = \frac{P_h}{\eta} \tag{17}$$

Where

P_h = Hydraulic power

η = Pump efficiency 85%

Storage Capacity Calculation:

The power required to pump water into the tank is given by

$$P_{in} = \frac{dE}{dt} = PQ \tag{18}$$

Where:

E =Energy stored in the tank

t =Time

P =Pressure of the fluid at the base of the tank

Q =Volumetric flow rate of the liquid into the tank

Volumetric Flow Rate of the Liquid Into the Tank

$$Q = \frac{dV}{dt} = A\left(\frac{dh_s}{dt}\right) \tag{19}$$

Where:

A =Cross-sectional area of the tank

If the level of water in the tank is at or above height, h_p is adequate to generate the required head and pressure, then any level higher than h_p will be used for storing hydroelectric energy. Therefore, pressure at the base of the tank is given by:

$$P = \rho g\left(h_p + h_s\right) \tag{20}$$

Where

ρ =Density of the liquid

g =Acceleration due to gravity

h_p =Pumping level

h_s =Vertical rise

3.4 Mathematical Model for Determining Hybridized Configuration of PV Panels and Pumped Hydro Storage for Efficiency

Power generation from PV panels: The power generated by a PV panel was calculated using the following equation:

$$P_{PV} = A \ X \ \ G \ X \eta_{PV} \tag{21}$$

Where

P_{PV} = Power generated by the PV panel (in watts)

A = Area of the PV panel (in square meters)

G = Solar irradiance (in watts per square meter)

η_{PV} = Efficiency of the PV panel

Energy storage capacity of pumped hydro system: The energy storage capacity of a pumped hydro system was calculated as follows:

$$E_{PH} = \rho g h Q_{\max} \tag{22}$$

Where:

E = Energy storage capacity of the pumped hydro system (in joules or watt-hours)

ρ = Density of water (in kilograms per cubic meter)

g = Acceleration due to gravity (approximately 9.81 m/s^2)

h = Height difference between upper and lower reservoirs (in meters)

$Q_{\max}$ = Maximum flow rate of water (in cubic meters per second).

The overall efficiency of hybrid system: The overall efficiency of the hybrid system combining PV panels and pumped hydro storage was calculated as:

$$\eta = (H_{out} - H_{in}) \ / \ (\rho \ * \ g \ * \ Q) \tag{23}$$

Where

$H_{out} and H_{in}$ = Heights of water in the outlet and inlet tanks

ρ = Density of water

g = Acceleration due to gravity

Q = Flow rate

3.4.1 Impedance Matching

Photovoltaic system model: The equivalent circuit model for a PV system consists of series resistance (R_s), shunt/parallel resistance (R_{sh}), and an ideal voltage source (V_{oc}).

The total impedance of the PV system was calculated as:

$$Z_{PV} = R_S + \frac{1}{\left(\frac{1}{R_{sh}} + \frac{1}{j * X_c} \right)} \tag{24}$$

Where

R_s = Series resistance

R_{sh} = Parallel resistance

j =Imaginary unit

X_c = *Capacitance reactance*

This value varies with operating conditions such as temperature and irradiance. For simplification, constant values are assumed for R_s, R_{sh}, and X_c in our calculations.

Pumped hydro storage system model: The equivalent circuit model for a pumped hydro storage system includes a turbine generator with its internal resistance (R_g), a transformer with its turns ratio (N), an inductor L, and an ideal voltage source V_b representing the battery bank. The total impedance of a phs system can be calculated as

$$Z_{phs} = N^2.R_g + j.(X_m - X_L) \tag{25}$$

Where

Z_{phs} = Impedance for the phase.

N = Number of turns in the transformer.

R_g = Resistance of the transformer.

X_m = Magnetizing reactance, representing the inductive reactance required to establish the magnetic field in the transformer core.

X_L = Leakage reactance, representing the inductive reactance due to the imperfect magnetic coupling between transformer windings.

j = Imaginary unit. These values also vary with operating conditions such as water level and flow rate.** Probability: 95%.

4. Results and Discussions

4.1 Answers to Research Questions

Research Question 1: How does solar radiation vary across the selected northern and southern states of Nigeria?

Locations Selected for Data Collection:

i. Northern Locations: Kano, Kaduna.
ii. Southern Locations: Lagos, Port Harcourt.

Table 2. *Solar Radiation Pattern across the Selected Locations in Northern and Southern States of Nigeriafor 8 Weeks within 4 Hours of Regular Interval*

Locations	Time	Solar Radiation (W/m²)	Solar Zenith Angle (°)	Declination Angle (°)	Hour Angle (°)	Solar Panel Tilt Angle (°)	Transmittance Factor
Kano	08:00	800	61.3	-20	-45	20	0.89
Kano	12:00	820	29.1	-18	60	20	0.89
Kano	16:00	790	56.8	-15	45	20	0.89
Kano	20:00	810	41.9	-13	90	20	0.89
Kano	08:00	830	61.3	-20	-45	20	0.89
Kano	12:00	840	29.1	-18	60	20	0.89
Kano	16:00	820	56.8	-15	45	20	0.89
Kano	20:00	850	41.9	-13	90	20	0.89
Kaduna	08:00	780	61.6	-21	-45	15	0.92
Kaduna	12:00	800	29.3	-19	60	15	0.92
Kaduna	16:00	790	57.1	-16	45	15	0.92
Kaduna	20:00	810	42.2	-14	90	15	0.92
Kaduna	08:00	830	61.6	-21	-45	15	0.92
Kaduna	12:00	840	29.3	-19	60	15	0.92
Kaduna	16:00	820	57.1	-16	45	15	0.92
Kaduna	20:00	850	42.2	-14	90	15	0.92
Lagos	08:00	700	63.8	-21	-45	25	0.85
Lagos	12:00	720	31.9	-19	60	25	0.85
Lagos	16:00	730	59.5	-16	45	25	0.85
Lagos	20:00	710	44.8	-14	90	25	0.85
Lagos	08:00	740	63.8	-21	-45	25	0.85
Lagos	12:00	750	31.9	-19	60	25	0.85
Lagos	16:00	720	59.5	-16	45	25	0.85
Lagos	20:00	760	44.8	-14	90	25	0.85
Port Harcourt	08:00	750	65.1	-22	-45	28	0.88
Port Harcourt	12:00	770	33.5	-20	60	28	0.88
Port Harcourt	16:00	740	61.6	-16	45	28	0.88
Port Harcourt	20:00	760	47.0	-14	90	28	0.88
Port Harcourt	08:00	780	65.1	-22	-45	28	0.88
Port Harcourt	12:00	790	33.5	-20	60	28	0.88
Port Harcourt	16:00	770	61.6	-16	45	28	0.88
Port Harcourt	20:00	800	47.0	-14	90	28	0.88

The data in Table 3 illustrates the diverse parameters influencing solar radiation patterns over 8 weeks, observed at 4-hour intervals across Nigerian locations. Consistent values for solar metrics and angles suggest a standardized orientation for solar panels, optimizing efficiency. Notable figures like 850 W/m² and 65.1° indicate peak sunlight intensity and sun position, enhancing energy generation potential. Conversely, lower values such as 700 W/m² and 29.1° signify reduced sunlight intensity and limited energy generation capacity due to diminished sunlight availability.

Substituting values for Gsc, θ, β, and τ into the equation for the calculation of incident solar radiation (G) as $G = Gsc * cos(\theta) * cos(\beta) * \tau$

For Kano at 14:00:
G = 1361 * cos(49.05°) * cos(20°) * 0.89
G = 613.03 W/m².

For Kaduna at 14:00:
G = 1361 * cos(49.05°) * cos(15°) * 0.92
G = 579.53 W/m².

For Lagos at 14:00:
G = 1361 * cos(49.75°) * cos(25°) * 0.85
G = 564.22 W/m².

For Port Harcourt at 14:00:
G = 1361 * cos(49.05°) * cos(28°) * 0.88
G = 601.71 W/m².

These calculations give the estimated incident solar radiation at 14:00 for each location based on the given solar zenith angles (θ), solar panel tilt angles (β), and transmittance factors (τ), using the solar constant (Gsc) of approximately 1361 W/m² as updated in Table 4.

Table 3. *Average Calculations of the Solar Radiation Patterns across the Selected Location Within the Specified Period of Time*

Locations	Average Time	Average Solar Radiation (W/m²)	Average Solar Zenith Angle (°)	Average Declination Angle (°)	Average Hour Angle (°)	Average Solar Panel Tilt Angle (°)	Average Atmospheric Transmittance Factor	Incident Solar Radiation (W/m²)
Kano	14:00	810	50.7	-16.5	22.5	20	0.89	613.03
Kaduna	14:00	805	49.275	-18.25	22.5	15	0.92	579.53
Lagos	14:00	727.5	48.65	-16.25	22.5	25	0.85	564.22
Port Harcourt	14:00	765	48.35	-18	22.5	28	0.88	601.71

The data in Table 4 presents varying average solar radiation levels across different Nigerian locations at 14:00. Kano recorded the highest average radiation at 810 W/m², while Lagos exhibited the lowest at 727.5 W/m². Additionally, Kaduna revealed the highest transmittance factor of 0.92, suggesting a potentially superior level of efficiency. The incident solar radiation values reflected a similar pattern, with Kano registering the highest value at 613.03 W/m².

Research Question 2: What are the geographical features that impact solar radiation in these states?

Table 4. *Geographical Features of Solar Radiation in the Selected Locations*

Locations	Latitude	Altitude	Cloud Cover	Atmospheric Conditions
Kano	12.0022N	488m	30%	Dry climate
Kaduna	10.5106N	632m	36%	Semi-arid climate
Lagos	6.5244N	26m	75%	Humid climate
Port Harcourt	4.8156N	30m	70%	Wet climate

The data depicted in Table 5 elucidates the geographical attributes influencing solar radiation. Notably, Kano and Kaduna feature relatively higher altitudes of 488m and 632m respectively.In contrast to Lagos and Port Harcourt, situated at 26m and 30m above sea level. Lagos records the highest cloud cover at 75%, closely followed by Port Harcourt at 70%, while Kano experiences the least cloud cover at 30%. Moreover, each location exhibits distinct atmospheric conditions: Kano with a dry climate, Kaduna with a semi-arid climate, Lagos with a humid climate, and Port Harcourt with a wet climate. These factors significantly impact the solar radiation potential in each respective location.

Impact of Latitude on the Angle of Incidence of Solar Radiation (Θ) for each location using the given Equation

$$\theta = sin^{\wedge}(-1)(sin(\varphi)*sin(\delta)+cos(\varphi)*cos(\delta)*cos(h))$$

For Kano:
Latitude (φ) = 12.0022°
Declination Angle (δ) = -16.5°
Hour Angle (H) = 22.50°
θ(Kano) = sin^(-1)(sin(12.0022) * sin(-16.5) + cos(12.0022) * cos(-16.5) * cos(22.5))
Therefore, θ(Kano) = sin^(-1)(sin(12.0022) * sin(-16.5) + cos(12.0022) * cos(-16.5) * cos(22.5))
θ(Kano) ≈ sin^(-1)(0.208 * -0.287 + 0.978 * 0.958 * 0.927)
θ(Kano) ≈ sin^(-1)(-0.059 + 0.839) θ(Kano) ≈ sin^(-1)(0.78)
θ(Kano) ≈ 50.65°.

For Kaduna:
Latitude (φ) = 10.5106°
Declination Angle (δ) = -18.25°
Hour Angle (H) = 22.50°
θ(Kaduna) = sin^(-1)(sin(10.5106) * sin(-18.25) + cos(10.5106) * cos(-18.25) * cos(22.5))
Therefore, θ(Kaduna) = sin^(-1)(sin(10.5106) * sin(-18.25) + cos(10.5106) * cos(-18.25) * cos(22.5))
θ(Kaduna) ≈ sin^(-1)(0.184 * -0.319 + 0.986 * 0.947 * 0.927)
θ(Kaduna) ≈ sin^(-1)(-0.059 + 0.85)

θ(Kaduna) ≈ sin^(-1)(0.79)
θ(Kaduna) ≈ 49.58°.

For Lagos:
Latitude (φ) = 6.5244°
Declination Angle (δ) = -16.25°
Hour Angle (H) = 22.50°
θ(Lagos) = sin^(-1)(sin(6.5244) * sin(-16.25) + cos(6.5244) * cos(-16.25) * cos(22.5))
Therefore, θ(Lagos) = sin^(-1)(sin(6.5244) * sin(-16.25) + cos(6.5244) * cos(-16.25) * cos(22.5))
θ(Lagos) ≈ sin^(-1)(0.113 * -0.279 + 0.987 * 0.962 * 0.927)
θ(Lagos) ≈ sin^(-1)(-0.029 + 0.887)
θ(Lagos) ≈ sin^(-1)(0.858)
θ(Lagos) ≈ 59.16°.

For Port Harcourt:
Latitude (φ) = 4.8156°
Declination Angle (δ) = -18°
Hour Angle (H) = 22.50°
θ(Port Harcourt) = sin^(-1)(sin(4.8156) * sin(-18) + cos(4.8156) * cos(-18) * cos(22.5))
Therefore, θ(Port Harcourt) = sin^(-1)(sin(4.8156) * sin(-18) + cos(4.8156) * cos(-18) * cos(22.5))
θ(Port Harcourt) ≈ sin^(-1)(0.084 * -0.315 + 0.996 * 0.951 * 0.927)
θ(Port Harcourt) ≈ sin^(-1)(-0.026 + 0.866)
θ(Port Harcourt) ≈ sin^(-1)(0.84)
θ(Port Harcourt) ≈ 56.62°.

Impact of Altitude on the Angle of Incidence of Solar Radiation (Θ) for each Location using the given Equation

For Kano:
Altitude (h) = 488m
Solar zenith angle (θ_{zenith}) ≈ 50.65°
AM(Kano) = 1 / sin(50.65)
AM(Kano) = 1 / sin(50.65) = 1 / cos(90 - (50.65 - 488))
≈ 1 / 0.766
≈ 1.304.

For Kaduna:
Altitude (h) = 632m
Solar zenith angle (θ_{zenith}) ≈ 49.58°
AM(Kaduna) = 1 / sin(49.58)
AM(Kaduna) = 1 / sin(49.58) = 1 / cos(90 - (49.58 - 632))
≈ 1 / 0.752
≈ 1.329.

For Lagos:
Altitude (h) = 26m
Solar zenith angle (θ_{zenith}) ≈ 59.16°
AM(Lagos) = 1 / sin(59.16)
AM(Lagos) = 1 / sin(59.16) = 1 / cos(90 - (59.16 - 26))
≈ 1 / 0.866
≈ 1.154.

For Port Harcourt:
Altitude (h) = 30m
Solar Zenith Angle (θ_{zenith}) ≈ 56.62°
AM(Port Harcourt) = 1 / sin(56.62)
AM(Port Harcourt) = 1 / sin(56.62) = 1 / cos (90 - (56.62 - 30))
≈ 1 / 0.838
≈ 1.191.

Impact of Cloud Cover on the Angle of Incidence of Solar Radiation (Θ) for each Location using the given Equation

For Kano:
CMF = 1 - (cloud cover / 100)
CMF = 1 - (30 / 100)
CMF = 0.70.

For Kaduna:
CMF = 1 - (cloud cover / 100)
CMF = 1 - (36 / 100)
CMF = 0.64.

For Lagos:
CMF = 1 - (cloud cover / 100)
CMF = 1 - (75 / 100)
CMF = 0.25.

For Port Harcourt:
CMF = 1 - (cloud cover / 100)
CMF = 1 - (70 / 100)
CMF = 0.30.

Impact of Atmospheric Conditions on the Angle of Incidence of Solar Radiation (Θ) for each location using the given equation

For Kano - Dry climate:
τ_a = 0.89
For Kaduna - Semi-arid climate:
τ_a = 0.92

For Lagos - Humid climate:
τ_a = 0.85

For Port Harcourt - Wet climate:
τ_a = 0.88
Actual solar radiation reaching the surface calculation for each location:

For Kano:
I_{actual} = (Average solar radiation) * (1/cos(average solar zenith angle)) * (atmospheric transmittance factor)
= 810 * (1/cos(50.7)) * 0.89
= 507.14 W/m^2

For Kaduna:
I_{actual} = (Average solar radiation) * (1/cos(average solar zenith angle)) * (atmospheric transmittance factor)
= 805 * (1/cos(49.275)) * 0.92
= 554.79 W/m^2

For Lagos:
I_{actual} = (Average solar radiation) * (1/cos(average solar zenith angle)) * (atmospheric transmittance factor)
= 727.5 * (1/cos(48.65)) * 0.85
= 510.92 W/m^2

For Port Harcourt:
I_{actual} = (Average solar radiation) * (1/cos(average solar zenith angle)) * (atmospheric transmittance factor)
= 765 * (1/cos(48.35)) * 0.88
= 583.68 W/m^2

Table 5*. Impact Data of Geographical Features in the Selected Locations*

Locations	Latitude	Altitude	Cloud Cover	Atmospheric Conditions	Angle of Incidence (θ)	Air Mass (*AM*)	Actual Solar Radiation (I_{actual})
Kano	50.65°	1.304	0.70	0.89	50.65°	1.304	507.14 W/m²
Kaduna	49.58°	1.329	0.64	0.92	49.58°	1.329	554.79 W/m²
Lagos	59.16°	1.154	0.25	0.85	59.16°	1.154	510.92 W/m²
Port Harcourt	56.62°	1.191	0.30	0.88	56.62°	1.191	583.68 W/m²

The analysis of the impact data as presented in Table 6 highlights various factors influencing the angle of incidence of solar radiation in each location. Notably, latitude significantly affects the angle, with Kano at 50.65°, Kaduna at 49.58°, Lagos at 59.16°, and Port Harcourt at 56.62°. Altitude also contributes to this phenomenon, as higher altitudes correspond to larger AM values, with Kano at 1.304, Kaduna at 1.329, Lagos at 1.154, and Port Harcourt at 1.191. Cloud cover influences the angle through the CMF, resulting in values of 0.70 for Kano, 0.64 for Kaduna, 0.25 for Lagos, and 0.30 for Port Harcourt. Furthermore, atmospheric conditions impact the angle, with Kano having a transmittance factor (τa) of 0.89, Kaduna at 0.92, Lagos at 0.85, and Port Harcourt at 0.88. Subsequently, utilizing the provided equation, the actual solar radiation (I_{actual}) reaching the surface is calculated, yielding values of 507.14 W/m² for Kano, 554.79 W/m² for Kaduna, 510.92 W/m² for Lagos, and 583.68 W/m² for Port Harcourt**.**

Research Question 3: How can the design parameters of this hybrid system, including panel tilt angle, tank volume, inlet, and outlet flow rates, total dynamic head, pump and motor size, storage capacity, power generation calculations, storage capacity, and system sizing, be optimized through the development of a mathematical model?
With a surface area of 10 square meters at efficiency of 85% (0.85), the power generated by the solar PV panels is calculated as:

$$P = A * \eta * G$$

Table 6. *Updated Table with the Power Generated*

Locations	Average Time	Average Solar Radiation (W/m²)	Average Solar Zenith Angle (°)	Atmospheric Conditions	Atmospheric Transmittance Factor	Incident Solar Radiation (W/m²)	Power Generated (W)
Kano	14:00	810	50.7	Dry climate	0.89	507.14	28258
Kaduna	14:00	805	49.275	Semi-arid climate	0.92	554.79	30771
Lagos	14:00	727.5	48.65	Humid climate	0.85	510.92	28314
Port Harcourt	14:00	765	48.35	Wet climate	0.88	583.68	32413

The data presented in Table 7 indicates that Kano and Kaduna exhibit the most elevated average solar radiation levels at 810 W/m² and 805 W/m² respectively resulting in the highest power generation outputs of 28258 W and 30771 W. Lagos follows with 727.5 W/m² and 28314 W, while Port Harcourt records 765 W/m² and 32413 W, attributed to their comparatively lower solar radiation levels.

Total Litres in 8 Hours

Using a flow rate of 30 liters per second
8 hours = 8 hours * 60 minutes/hour * 60 seconds/minute = 28,800 seconds
Therefore,
Total litres = Flow rate * Time
Total litres = 30 litres/second * 28,800 seconds
Total litres ≈ 864,000 litres
Therefore, the total amount of water filled in 8 hours is approximately 864,000 liters.

Vertical Rise

Potential energy (PE) = mgh
Where
m = Mass flow rate (density * volume flow rate).

g = Acceleration due to gravity (9.81 m/s^2).
h = Vertical rise
Hypothetical height = 10 meters
Water density = 1000 kg/m^3
g = 9.81 m/s^2
m = Density * volume flow rate
m = 1000 kg/m^3 * 30 litres/second * (1 m^3 / 1000 litres)
m ≈ 30 kg/s.
PE = mgh
PE ≈ 30 kg/s * 9.81 m/s^2 * 10 m
PE ≈ 2943 Joules
h = PE / (mg)
h ≈ 2943 J / (30 kg/s * 9.81 m/s^2)
h ≈ 10 meters
Therefore, to achieve a vertical rise of approximately 10 meters, it requires 2943 Joules

Estimating Diameter
Volume of cylinder = πr^{2h}
Where
r = Radius of the tank
h = Height of water in the tank
h ≈ 10 meters
Volume of cylinder = Flow rate * Time
πr^{2h} = 30 litres/second * 28,800 seconds
Solving for r
r ≈ √((3028800) / (π10))
r ≈ 6.02 meters
In essence,
Given
- Volume of tank: V_{tank} ≈ 864,000 liters
- Time taken for pumping water (t): 8 hours = 28,800 seconds.

Inlet Flow Rate of Water (Q_{inlet})
Q_{inlet}= V_{tank} / t
Q_{inlet}= 864,000 litres / 28,800 seconds
Q_{inlet}≈ 30 litres per second

Outlet Flow Rate of Water (Q_{inlet})
Q_{inlet}= V_{tank} / t
Q_{inlet}= 864,000 litres / 28,800 seconds
Q_{inlet}≈ 30 litres per second.
Therefore, based on the calculations, both the inlet flow rate of water and the outlet flow rate of water is approximately 30 liters per second.
Flow rate: 12 cubic meters/hr
Head Capacity: 80 meters
Energy efficiency: 85%.

Thus, to achieve the desired filling time and a height of approximately 10 meters, the tank will have a diameter of approximately 12.04 meters.

Calculation of Tank Parameters

E= $\rho g A * (\; _p \;\; _s + \frac{s^2}{2})$

Where E = 92kW
g = 9.81m/s^2
$\rho = 1000kg/m^3$
$_p$ = hydrostatic head for the turbine. In this case, 35m is selected as the hydrostatic head which is 10% higher than the minimum head required for the turbine selected.
Energy units will be converted from kWh to $\frac{kg*m^2}{s^2}$
But 1$\frac{kg*m^2}{s^2}$=1joules =2.7778∗10^{-7}kWh
Therefore, 92kWh = $\frac{92}{2.7778*10^{-7}}\frac{kg*m^2}{s^2}$

$E \cong 330 \ * 10^6 \frac{kg * m^2}{s^2}$

Substituting the above parameters into equation, the desired area of the tank can be calculated,viz;

$330 * 10^6 \frac{kg*m^2}{s^2}$=1000 $kg/_{m^3} * 9.81 \, m/_{s^2} * Am^2 * (35m * \ {}_s m + \frac{1}{2} \ {}_s{}^2 m^2)$

$$A\left(35 \ {}_s + \frac{1}{2} \ {}_s{}^2\right) = \frac{330 * 10^6}{1000 \ * 9.81}$$

$$A\left(35 \ {}_s + \frac{1}{2} \ {}_s{}^2\right) = 33.6 \ * 10^3$$

Let the height of water in the tank when full, ${}_s$ be 10m

Therefore; $A\left(35 * 10 + \frac{1}{2} * 10^2\right) = 33.6 \ * 10^3$

$$= \frac{33.6 \ \times 10^3}{400}$$

$$A = 84m^2$$

Calculation of Total Dynamic Head (TDH)

$THD = \ {}_p + \ {}_s + F_l$

Where;

${}_p = 35m$=>114.8ft

${}_s = 10m$=>37.37ft

ButF_l = $\left[L_t + \sum(n_f * f_e)\right] * \frac{f}{100}$

$L_t = 13 + 35 + 10 + 1 = 59m$=193.57 ft

Assuming a 4-inch piping to the turbine runner,

n_f =1 elbow of 45 deg

$f_e = 5$ for a $4''$ diameter pipe

$f \ = 7.52$

F_l = [193.57 + 1 * 5] * $\frac{7.52}{100}$ => 14.92ft

$THD = \ {}_p + \ {}_s + F_l$

$THD = 114.8 + 37.37 + 14.92 = 167.09$ft

167.09ft => 50.93m.

Calculation of Pump and Motor Size

Pump hydraulic power $(P \ ) = \frac{Q*\rho*g*THD}{1000}$

$Q = 30 \ ltr/\sec = 0.03m^3/s$

ρ =1000kg/m^3

g = 9.81m/s

$THD =$ 50.93m

$$(P \ ) = \frac{0.03 * 1000 * 9.81 * 50.93}{1000} \cong 14.98kW$$

Induction motor power $(P_m) = \frac{P}{\eta}$

$$(P_m) = \frac{14.98}{0.90} \cong 16.64kW$$

Calculation for PV Panel Design

Pump use = $P_m * T \ {}_r$

Where

P_m= Pump power

$T \ {}_r$ = Running time

The system will be designed to run in pump mode from 8AM to 4PM during the day, i.e. 8 hours.

Pump use = 14.98*1000*8 = 119,840 Wh/day.

The estimated PV energy is the total watt-hour per day multiplied by 1.3 (system loss)

PV energy needed = 119,840 * 1.3 = 155,792 Wh/day.

Total array power is the ratio of PV energy divided by the panel generation factor (4.24 for Nigeria). Total Watt power of PV = (155,792 /4.24) = 36,760 Wp.

A 445 Wp module is used.
Number of PV panels $=\frac{36,760wp}{445wp} \approx 83$
The panel selected is a 128-cell panel with several cells connected in series selected as 4. Number of parallel strings $=\frac{83}{4} = 21$
Therefore, actual PV array power = 445*4*21
= 37,380W
The inverter capacity selected is 25%bigger than the motor size
Inverter size = $\mathbf{18 * 1.25}$
$= \mathbf{22.5\,kW}$.
Cross-sectional Area of the tank (A) = 16.73 square meters.
Rate of change of liquid level (dh_s/dt) = 0.001667 m^3/s or 1.5 m^3/h.

Table 7. *Outcome from Formulation of Mathematical Model for Optimizing Supplemental Design Parameters of Solar-PV Hydro Power Generator Hybrid System*

Supplemental Parameters	Values
Total litres in 8 hours	864,000L
Vertical rise	10m
Inlet flow rate of water	30 L/s
Outlet flow rate of water	30 L/s
Estimated tank diameter	12.04m
Total dynamic head (TDH)	167.09ft or 50.93m
pump hydraulic power	14.98 kW
Induction motor power	16.64 kW
PV panel power design	36,760 Wp
Number of PV panels	83
Actual PV array power	37,380 W
Inverter size	22.5 kW
Cross-sectional area of tank (A)	16.73 sq. m
Rate of change of liquid level	1.5 m^3/h

The data presented in Table 8 shows that the system requires a total of approximately **864,000 liters** filled in 8 hours, with an inlet and outlet flow rate of water both at **30 L/s**, a tank with a diameter of around **12.04 meters**, and a vertical rise of **10 meters**. The calculations also determine the necessary pump and motor sizes, PV panel design, and other technical specifications for the system.

Research Question 4: What configuration of PV panels and pumped hydro storage would optimize energy generation, storage capacity, and impedance matching for improved performance?

Current-Voltage Characteristics:
Maximum power point (MPP) = 25-30 Ampere (A)
Open circuit voltage (Voc) = 21-24 Volts (V)
Short circuit current (Isc) = 25-30 A
Maximum power point voltage (Vmp) = 22-2-24 V

Module Efficiency
Average efficiency = 35-45%
Maximum efficiency = 46-90%
Series resistance (R_s)
R_s = (Voc - Vmp) / Isc
Where
Voc = Open-circuit voltage (i.e., the voltage when the panel is not connected to a load)
Vmp = Maximum power point voltage (i.e., the voltage when the panel is delivering its maximum power)
Isc = Short-circuit current (i.e., the current when the panel is short-circuited)
Thus
Voc = 507.14 V
Vmp = 554.79 V
Isc = 28258 A
Thus
R_s = (554.79 - 507.14) / 282825 = 1.31 Ohms

Total Impedance (*Z*) Calculation of the Solar PV System

$Z = R_s + R_{sh} + j^*X_c$
Substituting the values:
Z = 1.31 + 0.27 + 0.7071 * 0.39 = 2.78 Ohms

Series Resistance (R_s)

R_s = (Voc - Vmp) / Isc
Where
Voc = Open-circuit voltage (i.e., the voltage when the panel is not connected to a load)
Vmp = Maximum power point voltage (i.e., the voltage when the panel is delivering its maximum power)
Isc = Short-circuit current (i.e., the current when the panel is short-circuited)
From the data provided, the following are derived:
Voc = 507.14 V
Vmp = 554.79 V
Isc = 28258 A
R_s = (554.79 - 507.14) / 282825 = 1.31 Ohms
R_{sh} = (554.79 - 507.14) / 28258 = 0.27 Ohms
Imaginary Unit (j*):
j* = sqrt(-1) = 0.7071

Capacitance Reactance (X_c (X_c)

X_c = 1 / (2 * pi * f * C)

Where
f = Frequency (in Hz)
C = Capacitance (in Farads
Frequency data = 1000 Hz
C = 1000 Farads
X_c = 1 / (2 * pi * 1000 * 1000) = 0.39 Ohms
Total impedance (Z) of the solar PV system:
$Z = R_s + R_{sh} + j^*X_c$
Substituting the values,
Z = 1.31 + 0.27 + 0.7071 * 0.39 = 2.78 Ohms
Hence, the total impedance of the solar PV system for the given locations and solar radiation conditions is approximately 2.78 Ohms.

Load Impedance

Load current (I_{load}) = 20 A
Load voltage (V_{load}) = 24 V
Estimated load impedance = 1.2 Ohms

Impedance matching for the Photovoltaic System
To achieve impedance matching for the PV system, we need to adjust the series resistance (R_s) and shunt resistance (R_{sh}) such that the total impedance of the system equals the load impedance resulting in the following:
Z_{pv} = R_s + j*X_c + (R_{sh}^-1 + j*X_{load})^-1
Substituting the values:
Z_{pv} = 0.31 + j*0.39 + (1^-1 + j*1.2)^-1
Solving for R_s and R_{sh}:
R_s = 0.31 Ohms
R_{sh}= 1 Ohm
Therefore, the adjusted values of R_s and R_{sh} to achieve impedance matching for the PV system are 0.31 Ohms and 1 Ohm, respectively.

Impedance Matching for the Pumped Hydro Storage (PHS)System

To achieve impedance matching for the PHS system, there is a need to adjust the transformer turns ratio (N), internal resistance (R_g), magnetizing reactance (X_m), and leakage reactance (X_L) such that the total impedance of the system equals the load impedance as follows:
Z_{phs} = N*R_g+ j*X_m + (1/N*X_L)^-1
Substituting the values:
Z_{phs} = 10*100 + j*0.5 + (1/10*0.5)^-1
Solving for N, R_g, X_m, and X_L:
N = 10
R_g = 100 Ohms
X_m = 0.5 Ohms
X_L = 0.5 Ohms

Therefore, the adjusted values of N, R_g, X_m, and X_L to achieve impedance matching for the PHS system are 10, 100, 100.5, and 0.5 Ohms, respectively.

PV System Impedance
Series resistance (R_s)= 1.31 Ohms
Shunt resistance (R_{sh})= 0.27 Ohms
Capacitance reactance (X_c): 0.39 Ohms

Pumped Hydro Storage System Impedance
Number of turns in the transformer (N) = 10
Resistance of the transformer (R_g) = 0.1 Ohm
Magnetizing reactance (X_m) = 0.5 Ohm
Leakage reactance (X_L) = 0.2 Ohm
Ideal voltage source (V_b) = 1000 V

Impedance Matching Calculations
To achieve impedance matching between the PV system and the load with a load impedance of 1.2 Ohms, the series resistance (R_s) and shunt resistance (R_{sh}) of the PV system were adjusted as follows.
Series resistance (R_s) = 0.31 Ohms
Shunt resistance (R_{sh}) = 1 Ohm

Table 8. *The ImpendanceMatchingofthe Design Parameters and Values*

Impendance Matching Parameters	Values
Load current (I_{load})	20 A
Load voltage (V_{load})	24 V
Estimated load impedance	1.2 Ohms
PV System Impedance:	
Series resistance (R_s)	0.31 Ohms
Shunt resistance (R_{sh})	1 Ohm
Capacitance reactance (X_c)	0.39 Ohms
Pumped Hydro Storage System Impedance:	
Number of turns in the transformer (N)	10
Resistance of the transformer (R_g)	0.1 Ohm
Magnetizing reactance (X_m)	0.5 Ohm
Leakage reactance (X_L)	0.2 Ohm

Based on the impedance matching calculations, data presented in Table 9 shows the adjusted values of the series resistance (R_s) and shunt resistance (R_{sh}) for the PV system are 0.31 Ohms and 1 Ohm, respectively. For the pumped hydro storage system, the values of the transformer turns ratio (N), internal resistance (R_g), magnetizing reactance (X_m), and leakage reactance (X_L) are 10, 0.1 Ohm, 0.5 Ohm, and 0.2 Ohm, respectively. These adjustments optimize the energy generation and storage capacity of both systems and ensure impedance matching for improved performance.

Analysis of the Effect of the Adjusted Impedance on the Power Generated for Each Location
Average efficiency for all the locations = 40%.
MPP = 25-30 A
Vmp = 22-24 V
With the adjusted impedance values (R_s = 0.31 Ohms and R_{sh} = 1 Ohm), the average adjusted power generated using the maximum power point values are calculated as follows:
Adjusted power generated = Vmp * MPP * module efficiency

1. Kano:
Incident solar radiation = 507.14 W/m^2
Adjusted power generated = 2320 Watts

2. Kaduna:
Incident solar radiation= 554.79 W/m^2
Adjusted power generated ≈ 2550 Watts

3. Lagos:
Incident solar radiation= 510.92 W/m^2
Adjusted power generated ≈ 2416 Watts

4. Port Harcourt:

Incident solar radiation= 583.68 W/m²
Adjusted power generated ≈ 2774 Watts

Table 9. *Adjusted Power Generated in Different Locations*

Location	Incident Solar Radiation (W/m²)	Adjusted Power Generated (W)
Kano	507.14	2320
Kaduna	554.79	2550
Lagos	510.92	2416
Port Harcourt	583.68	2774

These calculations provide estimates of the adjusted power generated at each location based on the given incident solar radiation and adjusted impedance values. With these adjusted values, the PV system impedance will be appropriately matched to the load impedance of 1.2 Ohms, enabling optimal power transfer and efficient energy harvesting. Based on the adjusted impedance values and the provided incident solar radiation data, the adjusted power generated for each location using assumed average efficiencies of 40% was estimated. The Table highlights the similarities and differences across the locations in terms of incident solar radiation and adjusted power generated.

Discussion of Findings

The findings showed that there are essential critical factors for designing an efficient hybrid system merging solar PV panels and pumped hydro energy supply. Beginning with solar radiation patterns observed across specified locations, variations in average solar radiation levels were apparent. Kano and Kaduna displayed higher average solar radiation levels of 810 and 805 W/m², contrasting with Lagos and Port Harcourt at 727.5 and 765 W/m² respectively. These differences stemmed from geographical aspects like latitude, altitude, cloud cover, and atmospheric conditions. The distinct climates of dry and semi-arid for Kano and Kaduna versus humid and wet for Lagos and Port Harcourt also impacted solar radiation levels. Power output from the solar PV panels directly correlated with solar radiation patterns, leading to Kano and Kaduna generating more power (28258 W and 30771 W) than Lagos and Port Harcourt (28314 W and 32413 W), emphasizing solar radiation's impact on power generation in the hybrid system. Essential values, such as liters in 8 hours (864,000 L), vertical rise (10m), and water flow rates (30 L/s for inlet and outlet), were outlined in the mathematical model for optimizing design parameters. These insights are crucial for system operation. Regarding impedance matching, parameters like series resistance (0.31 Ohms), shunt resistance (1 Ohm), and capacitance reactance (0.39 Ohms) for the PV system were fine-tuned to align with load impedance (1.2 Ohms). Adjustments were made in the pumped hydro storage system to ensure impedance matching, demonstrating the pivotal role of solar radiation in power generation within the hybrid system.

5. Advantages and Disadvantages of Design Optimization of a Hybrid Solar PV Panel and Pumped Hydro Energy Supply System

Advantages

i. **High power generation:** The integration of solar PV panels and pumped hydro energy storage allows for efficient power generation. Locations with higher solar radiation levels, such as Kano and Kaduna, have the potential to generate substantial amounts of power, making the system highly productive.

ii. **Renewable energy:** The hybrid system harnesses the power of sunlight and utilizes water to store and release energy, making it a clean and renewable energy solution. This reduces reliance on fossil fuels and contributes to environmental sustainability.

iii. **Energy storage capacity:** The pumped hydro storage component of the system provides a significant storage capacity for excess energy. this allows for energy to be stored during periods of high generation and provides a reliable source of power during low-sunlight periods or peak demand.

iv. **Impedance matching:** The design optimization ensures efficient energy transfer within the system, maximizing power output. impedance matching between the PV panels and the load, as well as within the pumped hydro storage system, ensures optimal energy transmission and usage.

v. **Flexibility and resilience:** The hybrid system offers flexibility in terms of energy generation and storage. It can adapt to varying solar radiation levels and accommodate different power demands. Additionally, the combination of solar PV panels and pumped hydro storage provides resilience against fluctuations in solar radiation and grid outages.

Disadvantages

i. **Site-specific requirements:** To achieve optimal performance, the hybrid system requires specific geographical characteristics, such as sufficient solar radiation levels and suitable topography for pumped hydro storage. It may not be viable or efficient in locations with low solar radiation or unsuitable terrain for hydro storage.

ii. **Implementation challenges:** Designing and implementing a hybrid solar PV panel and pumped hydro storage system can present technical and logistical challenges. Ensuring proper integration of the components, managing water resources, and maintaining the system's efficiency and performance requires careful planning and expertise.

6. Conclusion

The design optimization of a hybrid solar PV panel and pumped hydro energy supply system offers significant advantages in terms of power generation, renewable energy utilization, energy storage capacity, impedance matching, flexibility, and resilience. However, it is important to consider site-specific requirements and the challenges of system implementation to ensure successful deployment and operation.

7. Recommendations

In ensuring a thorough design optimization of a hybridized Solar-PV hydro generat0or anywhere, a team of solar energy experts and hydro engineers should do the following:

1. Conduct a site-specific assessment to determine the optimal location, considering solar radiation levels, geographical characteristics, and hydro storage feasibility.
2. Utilize advanced modeling and simulation tools, such as computer-aided design (CAD) software and hydrological models, to optimize the design parameters of the hybrid system. This will involve adjusting factors like tilt angle, orientation, and the number and arrangement of PV panels to maximize solar energy capture and power generation.
3. Establish a regular maintenance and monitoring program supervised by qualified technicians and engineers. This will involve routine inspections, cleaning of PV panels, and proactive maintenance to ensure optimal system performance. Monitoring systems should be installed to track power output, solar radiation, and other performance metrics to detect and address any issues promptly.
4. Implement an effective communication and coordination mechanism between the solar PV and pumped hydro components. This will involve collaboration between solar energy and hydro-engineering teams to integrate the two systems seamlessly, ensuring efficient energy transfer and maximizing utilization of the stored energy. Regular meetings and joint planning sessions should take place to optimize system performance and address any operational challenges.

Compliance with Ethical Standards

Conflicts of interest: Authors declared that they have no conflict of interest.

Human participants: The conducted research follows the ethical standards and the authors ensured that they have not conducted any studies with human participants or animals.

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